EJEMPLOS DE TRANSFORMADA Z INVERSA 2024-2

1) Dada la ecuación en diferencias de un sistema discreto:
   y(n) = x(n) -y(n-1)/6 + y(n-2)/6
   Calcular:
   a) H(z) del sistema.
   b) h(n) del sistema por cuatro métodos

APLICANDO TZ:

$$Y(z) = X(z) - \frac{z^{-1}}{6}Y(z) + \frac{z^{-2}}{6}Y(z)$$

$$Y(z)\left\{1 + \frac{z^{-1}}{6} - \frac{z^{-2}}{6}\right\} = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + z^{-1}/6 - z^{-2}/6} = \frac{1}{(1+z^{-1}/2)(1-\frac{z^{-1}}{3})}$$

$P_1 = -1/2$

$P_3 = 1/3$

$$= \frac{z^2}{z^2 + z/6 - 1/6} = \frac{z^2}{(z+\frac{1}{2})(z-\frac{1}{3})}$$

$$\begin{cases} \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6} \\ \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6} \end{cases}$$

1) POR DEFINICIÓN Y CAUCHY:

RECORDANDO:

$$h(n) \triangleq TZI\{H(z)\} = \frac{1}{2\pi j}\oint_c H(z)\, z^{n-1}\, dz$$

POR CAUCHY:

$$\frac{1}{2\pi j}\oint_c z^{n-1-m}\, dz = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

$$\frac{1}{2\pi j}\oint_c \frac{F(z)}{z - z_0}\, dz = \begin{cases} F(z_0) & \text{si } z \in C \\ 0 & \text{si } z \text{ fuera de } C \end{cases}$$

$$\frac{1}{2\pi j}\oint \frac{F(z)}{(z-z_0)^k}\, dz = \begin{cases} \frac{1}{(k-1)!}\left.\frac{d^{k-1}F(z)}{dz^{k-1}}\right|_{z=z_0} & z_0 \in C \end{cases}$$

$$\frac{1}{2\pi j}\oint_c \frac{F(z)}{G(z)}\, dz = \frac{1}{2\pi j}\oint \sum_{i=1}^{p} \frac{A_i(z)}{z - z_i}\, dz = \sum_{i=1}^{p} A_i(z_i)$$

$$A_i(z) = (z - z_i)\frac{F(z)}{G(z)}$$

APLICANDO DEFINICIÓN Y CAUCHY:

$$h(n) = \frac{1}{2\pi j} \oint_c \frac{z^2}{(z+\frac{1}{2})(z-\frac{1}{3})} z^{n-1} dz = \left.\frac{z^n z}{z - 1/3}\right|_{z=-1/2} + \left.\frac{z^n z}{z + 1/2}\right|_{1/3}$$

$$\frac{3-2}{6} = \frac{1}{6} \qquad \frac{2+3}{6} = 5/6$$

$$\Rightarrow h(n) = \frac{3}{5}(-1/2)^n + \frac{2}{5}(1/3)^n \, u(n)$$

2) POR FRACCIONES PARCIALES:

$$H_1(z) = \frac{H(z)}{z} = \frac{z}{(z+1/2)(z-1/3)} = \frac{A}{z+1/2} + \frac{B}{z-1/3}$$

$$A = H_1(z)(z+1/2)\Big|_{-1/2} = \frac{z}{z-1/3} \cdot \frac{-1/2}{-1/2-1/3} = \frac{1/2}{\frac{5}{6}} = 3/5$$

$$B = H_1(z)(z-1/3)\Big|_{1/3} = \left.\frac{z}{z+1/2}\right|_{1/3} = \frac{1/3}{\frac{1}{3}+\frac{1}{2}} = \frac{1/3}{5/6} = \frac{2}{5}$$

$$\Rightarrow \quad H(z) = \frac{3/5\, z}{z+1/2} + \frac{2}{5}\frac{z}{z-1/3} \Rightarrow \text{DE TABLAS}$$

$$h(n) = \left\{\frac{3}{5}(-1/2)^n + \frac{2}{5}(1/3)^n\right\} u(n)$$

3) POR DIVISIÓN DE POLINOMIOS Y TZ:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + z^{-1}/6 - z^{-2}/6} = C(z) + \frac{R(z)}{D(z)}$$

$$\begin{array}{r|l}
1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2} & 1 - \frac{1}{6}z^{-1} + \frac{7}{36}z^{-2} \\
\hline
 & 1 \\
 & -\left(1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right) \\
\hline
 & -\frac{1}{6}z^{-1} + \frac{1}{6}z^{-2} \\
 & -\left(-\frac{1}{6}z^{-1} - \frac{1}{36}z^{-2} + \frac{1}{36}z^{-2}\right) \\
\hline
 & \frac{7}{36}z^{-2} - \frac{1}{36}z^{-2} \\
 & \vdots
\end{array}$$

IGUALANDO COEFICIENTES DE DEFINICIÓN DE TZ

$$h(n) = \left\{1, -\frac{1}{6}, \frac{7}{36} \ldots \right\}$$

4) METODO COMPUTACIONAL, CON $x(n)=\delta(n)$, $h(-1)=0$, $h(-2)=0$

$$y(n) = x(n) - \frac{1}{6}y(n-1) + \frac{1}{6}y(n-2)$$

$$\Rightarrow h(0) = \delta(0) - \frac{1}{6}y(-1) + \frac{1}{6}y(-2) = 1$$

$$h(1) = \delta(1) - \frac{1}{6}y(0) + \frac{1}{6}y(-1) = -\frac{1}{6}$$

$$h(2) = \delta(2) - \frac{1}{6}y(1) + \frac{1}{6}y(0) = -\frac{1}{6}\left(-\frac{1}{6}\right) + \frac{1}{6} = \frac{1}{36} + \frac{1}{6} = \frac{7}{36}$$